Exploring Space Through ALGEBRA

EDUCATOR EDITION

Algebra I and Geometry

Lunar Rover

Instructional Objectives

Students will

- create a scale drawing to model a real life problem;
- apply the Pythagorean Theorem and distance/rate formula ($d = rt$); and
- analyze data to find a solution.

Prerequisites

Prior to this activity, students should have experience applying formulas. Students should be familiar with using calculators and evaluating formulas, as well as have a basic knowledge of scale factor and application of the Pythagorean Theorem.

Background

This problem is part of a series that applies mathematical principles to the U.S. Space Exploration Policy.

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. It seeks answers to fundamental questions about our existence, responds to recent discoveries and puts in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown, we learn and we understand. As we take our first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

The vision for space exploration includes returning the space shuttle safely to flight, completing the International Space Station, developing a new exploration vehicle and all the systems needed for embarking on extended missions to the Moon, Mars, and beyond.

In 1971 the Apollo 15 mission was the first to carry a lunar roving vehicle (LRV). This LRV (Figure 1) allowed astronauts to travel farther from their landing sites than in previous missions. This enabled them to explore and sample a much wider variety of lunar materials.

Because the vehicle was unpressurized, its longest single trip was 12.5 kilometers (7.8 miles). Its maximum range from the Lunar Module (LM) was 5.0 kilometers (3.1 miles). LRVs were also on Apollo 16 (1972) and Apollo 17(1972).

If the LRV should happen to fail at any time during the extravehicular activity (EVA), the astronauts must have sufficient life support

Grade Level
8-12

Subject Area
Mathematics: Algebra I, Geometry

Key Concept
Pythagorean Theorem, distance/rate formula ($d = rt$)

Teacher Prep Time
5-10 minutes

Problem Duration
45-60 minutes

Technology
TI-83 Plus family; TI-84 Plus family; TI Explorer; other scientific calculator, spreadsheet software

Materials
- Student Edition
- Ruler or straight edge
- Graph paper
- Colored pencils

Degree of Difficulty
Moderate to Difficult

Skill
Calculator use; solving problems in a geometric context; creating data tables; applying formulas

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NCTM Principles and Standards
- Algebra
- Geometry
- Problem Solving
- Communication
- Representation

consumables to be able to walk back to the LM. This distance is called the “walkback limit”, and it was approximately 10 kilometers (about 6 miles). Because of the reliability of the LRV and of the spacesuits, this restriction was relaxed on Apollo 17 for the longest traverse from the landing site, about 20 km (about 12 miles).

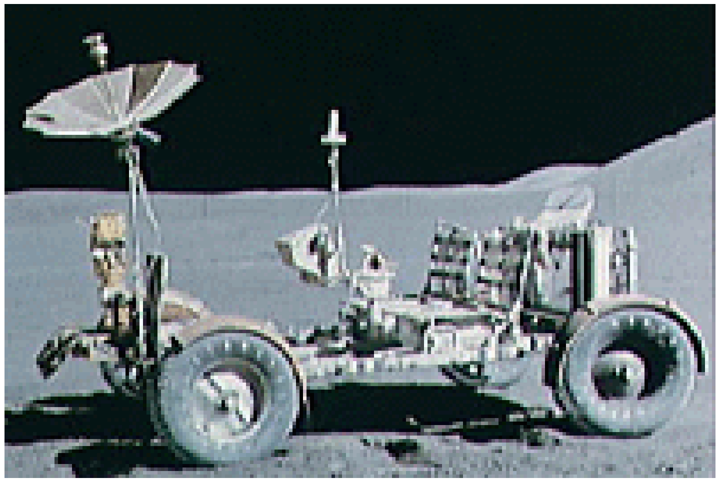

Figure 1: Apollo 15 Lunar Roving Vehicle taken on the Moon (NASA)

When NASA returns to the Moon, it will once again take LRVs to extend the distance astronauts may travel over the Moon’s surface. Modern unpressurized rovers (Figure 2) will look similar to those of the Apollo years and steer like a car. These vehicles will be limited to local travels of 10 to 20 kilometers (about 6 to12 miles) from the outpost site during short periods of time that are less than 10 hours. Astronauts will still need to wear space suits while traveling in these unpressurized LRVs.

Figure 2: Unpressurized Rovers (NASA concept)

A second type of LRV will be pressurized and will have about 200 km as its starting point. Pressurized rovers (Figure 3) will give astronauts the ability to travel long distances and perform extended science missions away from their habitat. These pressurized surface vehicles will provide a comfortable indoor environment from which the crew can drive the rover and control a variety of sensing and manipulation tools. This allows exploration and science to be performed without the need to exit the vehicle. Vehicle concepts also include docking ports for the crew to directly enter the rovers from habitats and an airlock to allow EVA. The versatility of the vehicle, its power system, and its life support system will allow the astronauts to spend multiple Earth days away from their habitat performing work.

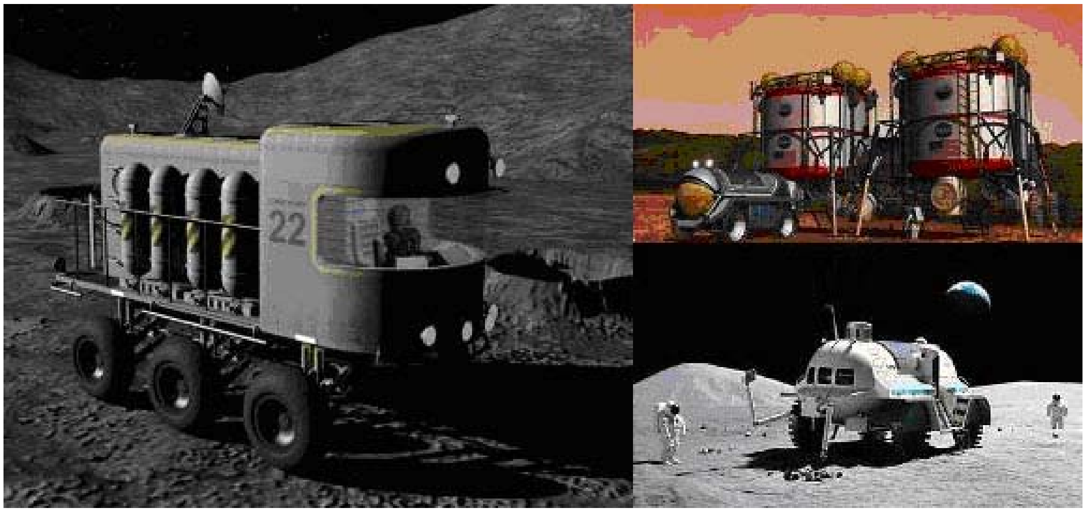

Figure 3: Pressurized Rovers (Artist concepts)

For more information about lunar roving vehicles and the U.S. Space Exploration Policy, visit www.nasa.gov.

NCTM Principles and Standards

Algebra

- Generalize patterns using explicitly defined and recursively defined functions.
- Use symbolic Algebra to represent and explain mathematical relationships.
- Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
- Draw reasonable conclusions about a situation being modeled.

Geometry

- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.
- Use geometric models to gain insights into, and answer questions in, other areas of mathematics.

Problem Solving

- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.

Communication

- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

Representation

- Create and use representations to organize, record, and communicate mathematical ideas.

Problem

You are on the mission planning team that will determine the best route for the crew to use on the first trip using the new pressurized LRV (Rover 1). This exploration mission will include gathering rock samples from around the Moon's deGerlache crater.

Rover 1 is at habitat A on the map near Shackleton crater. Its mission for the day is to drive to the rim of deGerlache crater to collect rock samples. Rocks can be collected at any point along the edge of the crater (along segment $\overline{PQ}$ on the map). Before Rover 1 and the crew return to the habitat they must also stop at location B on the map in order to reset a seismic sensor that has been gathering data about the interior of the Moon. All distances are denoted on the problem diagram.

1. Minimal Path Problem: the students must find the shortest total path for the pressurized Rover 1 to travel for this exploration.
2. Minimal Time Problem: the students must find the best location for Point C, so that the total travel time for the distance AC + CB is minimized,
3. Find the minimum using a graphing calculator.

Lesson Development

This activity focuses on the concept of minimizing distance and time. Students will use formulas to calculate the total distance and the total time for a particular task. They will then use the data and problem solving skills to determine the minimal distance and time required.

Students will work in groups of three or four to make a scale drawing of the mission. Students will choose several locations along the crater at which to stop. Based on this scale drawing, students will find the distances between the locations using the Pythagorean Theorem. They will make a table to capture their data. Encourage each member of the group to choose different locations and then to combine their data. Students will then analyze their data to solve the minimal path and minimal time problems.

Wrap-Up

Once students are satisfied that they have solved the problem, have groups share their solutions with each other. They should note any differences in their solutions and discuss the possible reasons for those differences.

Extensions

Students can use spreadsheet software to create tables of their data.

Solution Key

You are on the mission planning team that will determine the best route for the crew to use on the first trip using the new pressurized LRV (Rover 1). This exploration mission will include gathering rock samples from around the Moon's deGerlache crater.

Rover 1 is at habitat A on the map near Shackleton crater. Its mission for the day is to drive to the rim of deGerlache crater to collect rock samples. Rocks can be collected at any point along the edge of the crater (along segment $\overline{PQ}$ on the map). Before Rover 1 and the crew return to the habitat they must also stop at location B on the map in order to reset a seismic sensor that has been gathering data about the interior of the Moon. All distances are denoted on the problem diagram (Figure 4).

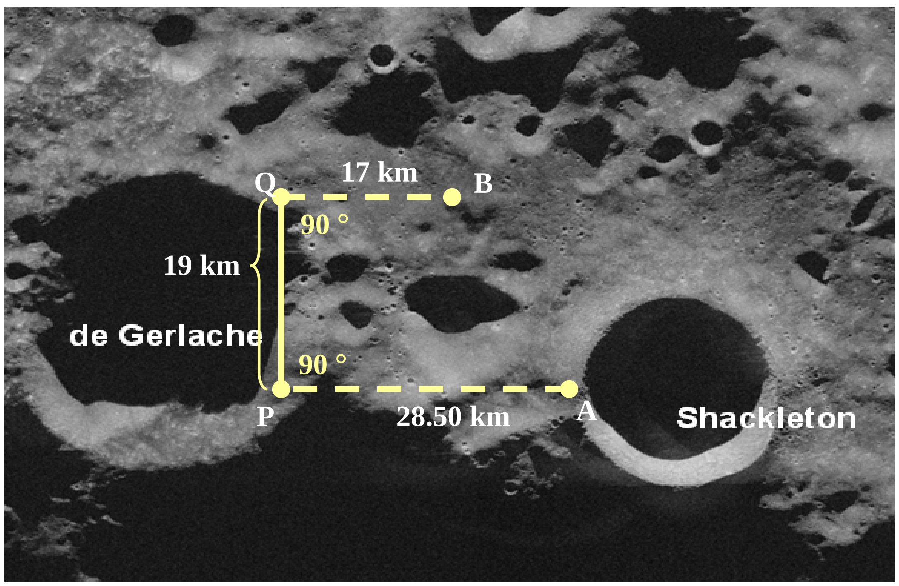

Figure 4: Problem diagram

1. **Minimal Path Problem**

   Your mission planning team must find the shortest total Path for the pressurized Rover 1 to travel for this exploration. The crew will start at habitat A and travel to a point along deGerlache crater (on segment $\overline{PQ}$) to collect rock samples. This point will be denoted as point C. Then, the crew will take Rover 1 from that point C to location B to reset the seismic sensor.

   a. Create a scale drawing of the mission on the graph provided (Graph 1). Point P is located at the origin, segment $\overline{AP}$ lies on the x-axis, and segment $\overline{PQ}$ lies on the y-axis. Chose a point on $\overline{PQ}$ along the deGerlache Crater that you think would give the shortest total distance. Label that point X.

   b. Using a colored pencil and a straight edge, plot the point, C (0,3), on segment $\overline{PQ}$, along the deGerlache crater, where the crew will gather rock samples. Draw the path of Rover 1 from A to C, then to B. Label your points.

   Models will vary.

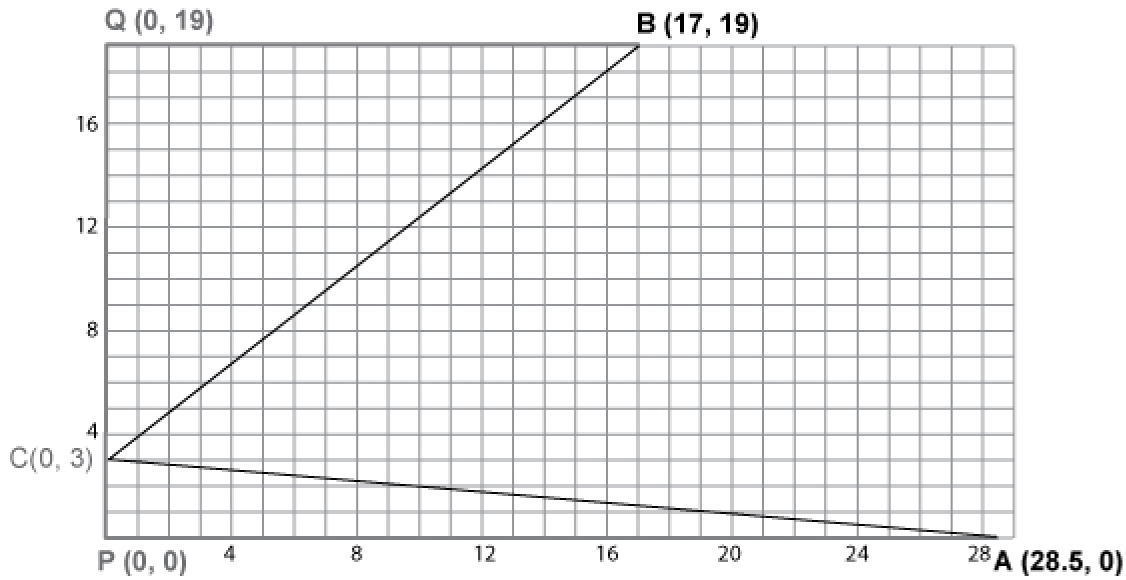

Figure 5: Possible mission graph with the location of C graphed

c. Find the total length AC + CB. You will need to make several calculations using the Pythagorean Theorem.

1. Working with your mission planning team, plot 4 different locations from (0,1) to (0,19) for Point C (chose at lease one point with a fractional value for y) and draw the paths on your mission graph.
2. Enter the corresponding data in the Minimal Path Table (Table 1) including a detailed process for finding each calculation. Round your answers to the nearest hundredth. The work for the given point, C(0,3) is already shown in the table. For the general point, (0, n), each entry in the last row will be a variable expression instead of a number. Use the process in the previous rows to draw conclusions about the entries in the last row.

Tables will vary.

Table 1: Minimal Path Table

C (0, *n*)	**PC**	**CQ**	**AC**	**CB**	**AC+CB**
(0, 3)	3	19 – 3 = 16	$\sqrt{28.50^2+3^2}=28.66$	$\sqrt{17^2+16^2}=23.35$	28.66 + 23.35 = 52.01
(0, 7)	7	19 – 7 = 12	$\sqrt{28.50^2+7^2}=29.35$	$\sqrt{17^2+12^2}=20.81$	29.35 + 20.81 = 50.16
(0, 11)	11	19 – 11 = 8	$\sqrt{28.50^2+11^2}=30.55$	$\sqrt{17^2+8^2}=18.79$	30.55 + 18.79 = 49.34
(0, 15)	15	19 – 15 = 4	$\sqrt{28.50^2+15^2}=32.21$	$\sqrt{17^2+4^2}=17.46$	32.21 + 17.46 = 49.67
(0, 19)	19	19 – 19 = 0	$\sqrt{28.50^2+19^2}=34.25$	$\sqrt{17^2+0^2}=17.00$	34.25 + 17.00 = 51.25
(0, *n*)	*n*	19 – n	$\sqrt{28.50^2+n^2}$	$\sqrt{17^2+(19-n)^2}$	$\sqrt{28.50^2+n^2}+\sqrt{17^2+(19-n)^2}$

d. Working with your mission planning team, analyze your table. According to your data, where is the best location for C so that the total distance Rover 1 travels is minimized?

The minimal distance is found when C is the point (0,12) or when C is 12 km from Point P on $\overline{PQ}$.

What is the distance traveled?

49.31 km.

Compare your results with those of 2 other teams. Does there seem to be one best location in order to minimize the path?

Yes, there will be an interval around C = (0, 12) where the distance will be minimized. Encouraging students to use fractional values for their choice of C will demonstrate this.

e. Show your solution on your mission graph by highlighting Rover 1's path for the shortest distance traveled. How close was your prediction, X, for the shortest total distance to the actual value found?

The solution for minimal distance is represented by the heavy line. Answers will vary for how close their prediction was.

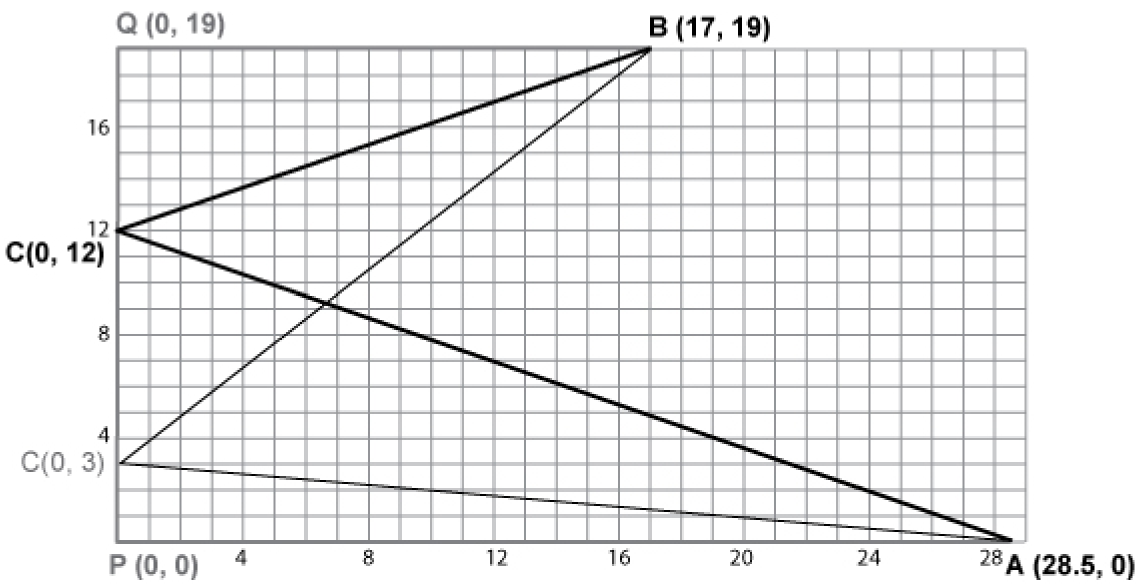

Figure 6: Possible solution for minimal distance shown on a mission graph.

2. **Minimal Time Problem**

The average speed of Rover 1 is 8.9 km/hr with an empty payload. With a full payload of rocks the average speed drops to 5.5 km/hr. Find the best location for point C so that you minimize the total travel time for the distance AC + CB. Recall that *d = rt, and t = d/r.*

a. Using the locations chosen in Problem 1 and the Minimal Time table below (Table 2), find the time needed for each path. Include in the last column of the table a detailed process for finding each calculation.

Example: The time from A to C is 28.66 / 8.90, and the time from C to B is 23.35 / 5.50.
The total time is 28.66 / 8.90 + 23.35 / 5.50 = 7.47

Table 2: Minimal Time Table

C (0, *n*)	**AC**	**CB**	**Total Time**
(0, 3)	28.66	23.35	28.66 / 8.90 + 23.35 / 5.50 = 7.47
(0, 7)	29.35	20.81	29.35 / 8.90 + 20.81 / 5.50 = 7.08
(0, 11)	30.55	18.79	30.55 / 8.90 + 18.79 / 5.50 = 6.85
(0, 15)	32.21	17.46	32.21 / 8.90 + 17.46 / 5.50 = 6.79
(0, 19)	37.25	17.00	37.25 / 8.90 + 17.00 / 5.50 = 7.28
(0, *n*)	$\sqrt{28.50^2 + n^2}$	$\sqrt{17^2 + (19 - n)^2}$	$\frac{\sqrt{28.50^2 + n^2}}{8.90} + \frac{\sqrt{17^2 + (19 - n)^2}}{5.50}$

b. According to your data, where is the best location for C so that the total time traveled is minimized? What is the minimum time for the mission?

Time is minimized when C is between the points (0, 14) and (0, 15) or when C is between 14 and 15 km from P on $\overline{PQ}$. *The minimum time is 6.79 hours.*

c. Show your solution on your mission graph by highlighting Rover 1's path for the minimum travel time.

The solution for minimal time is represented by the dashed line.

d. Based on your work so far, where do you think is the best place to collect rocks along the deGerlache Crater in order to minimize both distance and time? Explain.

Distance and time should both be minimized around (0,13). This is the point halfway between the minimum distance and the minimum time.

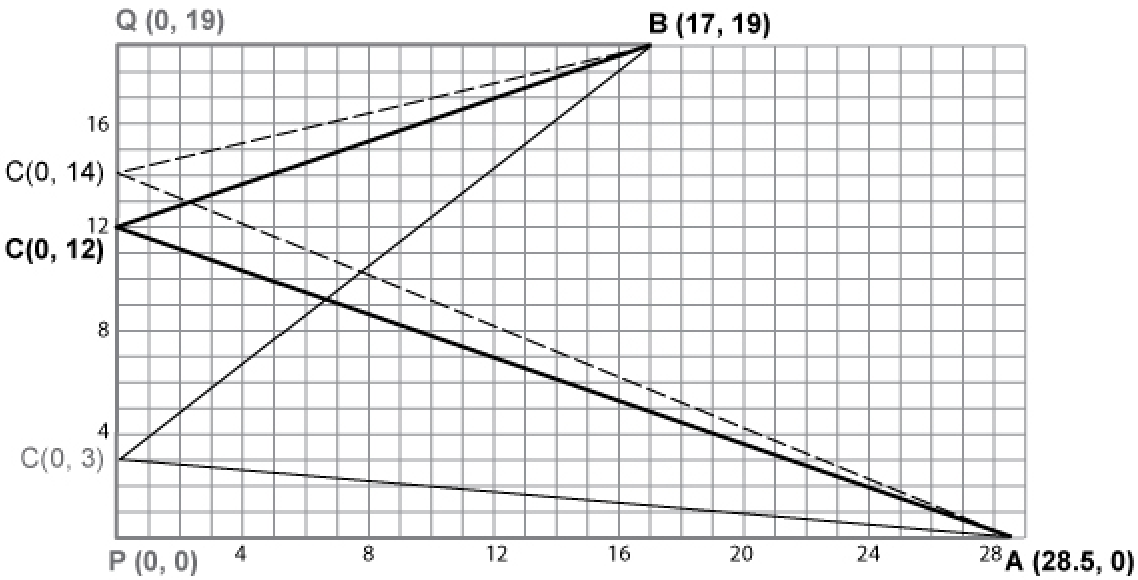

Figure 7: Possible solution for minimal time shown on a mission graph.

3. **Find the minimum using a graphing calculator.**

   a. Given any point C(0, *n*) along segment $\overline{PQ}$, write an expression in terms of *n* to represent the following quantities. Use your process from the Minimal Path table in Problem 1.

   PC = n

   CQ =19 - n

   $AC = \sqrt{28.50^2 + n^2}$

   $CB = +\sqrt{17^2 + (19 - n)^2}$

   Total Distance (AC + CB) $= \sqrt{28.50^2 + n^2} + \sqrt{17^2 + (19 - n)^2}$

   b. Enter the equation for the total distance into y1 in your graphing calculator. Graph the equation and use the minimum function of the calculator to approximate the minimum distance. Does it match your solution?

   Hint: To set the WINDOW values of the min and max of *x*, recall that *x* represents the distance, *n*, from P to C (first column). For the min and max values of *y*, recall that *y* represents the total distance, AC + CB (last column).

```
WINDOW
 Xmin=0
 Xmax=20
 Xscl=2
 Ymin=45
 Ymax=55
 Yscl=1
 Xres=1
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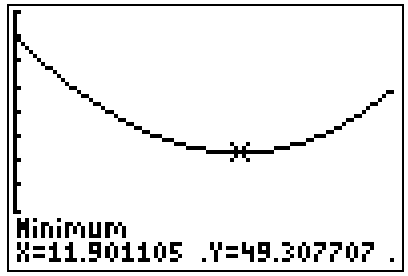


The minimum is when C is 11.90 km from point Q, which is very close to the answer of 12 found in Problem 1. The minimum distance of 49.31 km matches the solution found in Problem 1.

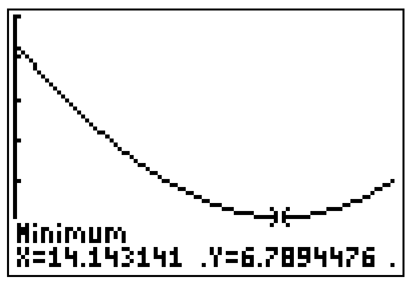
c. Using your Minimal Time table from Problem 2, write an equation to represent the total time.

$$t = \frac{\sqrt{28.50^2 + n^2}}{8.90} + \frac{\sqrt{17^2 + (19 - n)^2}}{5.50}$$

d. Repeat Part b, above, for the equation for total time. Does it match your solution?

Hint: The WINDOW values for the min and max of *x* are the same as above. For the min and max values of *y*, recall that *y* represents the total time (last column).

```
WINDOW
 Xmin=0
 Xmax=20
 Xscl=2
 Ymin=6.5
 Ymax=8
 Yscl=.25
 Xres=1
```



The minimum time is 6.79 hours and occurs when C is 14.14 km from point P. This matches the answer in Problem 2.

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Thanks to the subject matter experts for their contributions in developing this problem:

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